Repeated Division

David W. Carraher

Mar. 1, 2010

Quantitative Thinking

- Focus 1: The **linear density** of numbers and physical quantities

  - How <u>tightly packed</u> are they (when arranged in order)?

- Focus 2: **Mental operations on magnitudes**

  - How can numbers and quantities be composed and decomposed into other assemblages?

Let's start with the number 2. What number is half of 2?

If child says incorrect answer, probe by asking: *Could it be 1?* If child says "no", then stop.

Now, what number (fraction) is half of 1?

If child says incorrect answer, probe by asking: *Could it be $\frac{1}{2}$?*
If child says "no", then stop.

Is there a number (fraction) that is half of $\frac{1}{2}$? (Just look for yes or no; don't worry about what it is.)

OK. Is there another number (fraction) that is half of that?

Could we keep going? That is, for every number (fraction) we get, is there another number (fraction) that is half of it?

Why or why not?

As we do this, are the numbers (fractions) getting bigger or smaller?

How do you know?
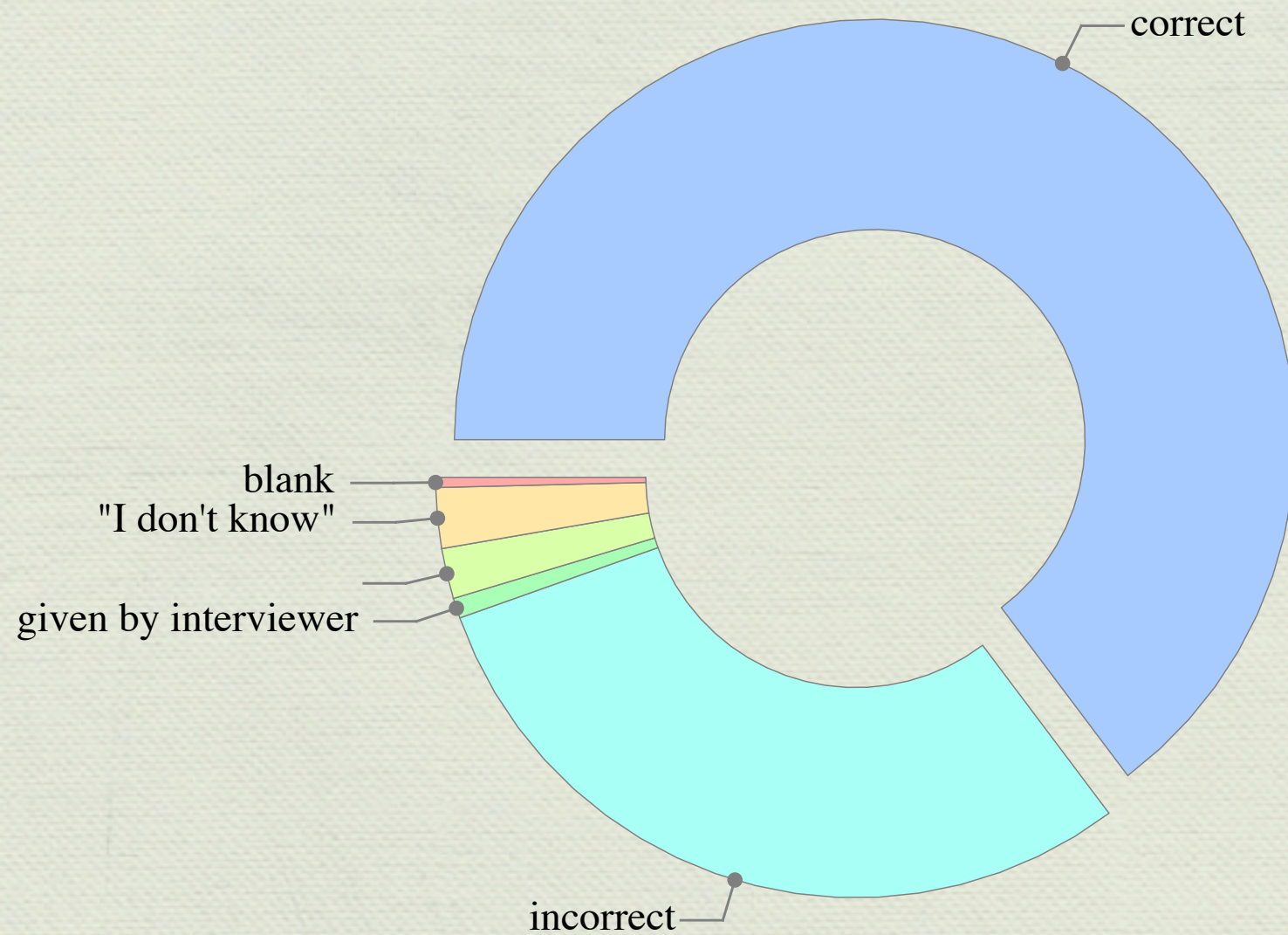
Would we ever get zero as our answer?

Why or why not?

Repeated Halving

- What number is one-half of two?

- What number is one-half of one?

- Is there a number that is one-half of one half?

- Is there a number that is one-half of that?

What number is half of two?

correct

blank
"I don't know"

given by interviewer

incorrect

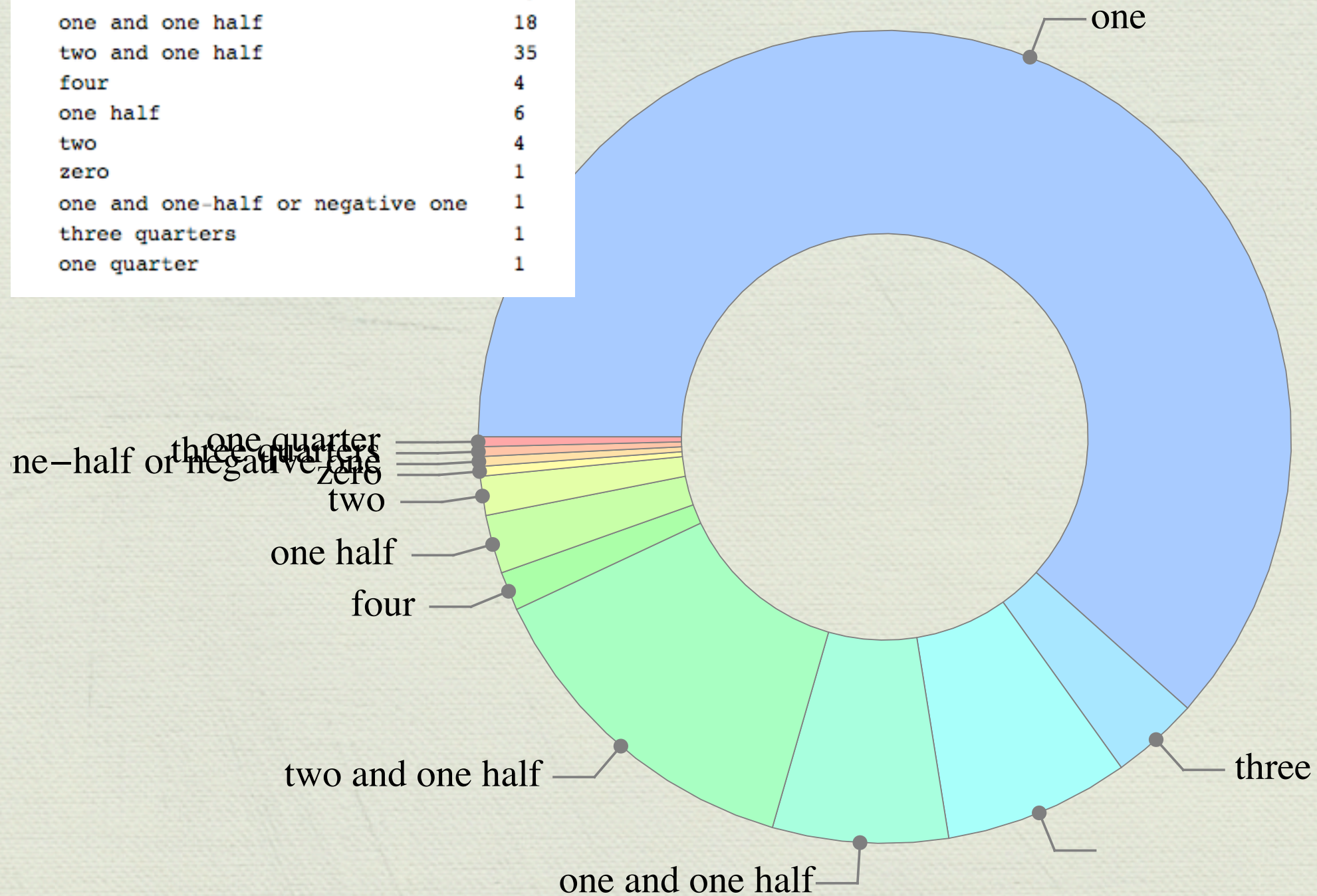| data1 // TableForm	
correct	167
incorrect	77
given by interviewer	2
	5
"I don't know"	6
blank	1

What number is half of two?


```
▼ data11 // TableForm
  one                                  159
  three                                9
                                       19
  one and one half                     18
  two and one half                     35
  four                                 4
  one half                             6
  two                                  4
  zero                                 1
  one and one-half or negative one     1
  three quarters                       1
  one quarter                          1
```
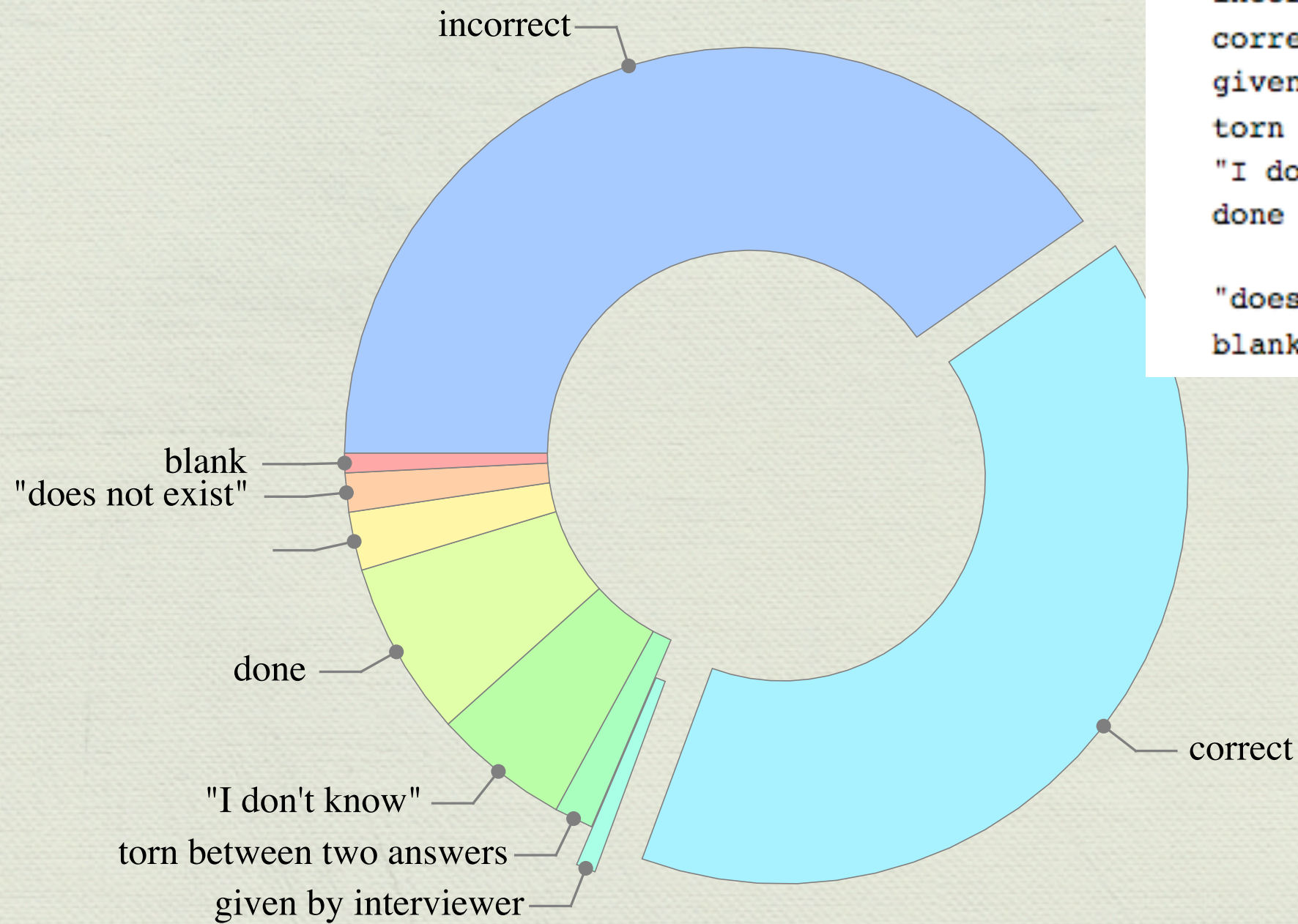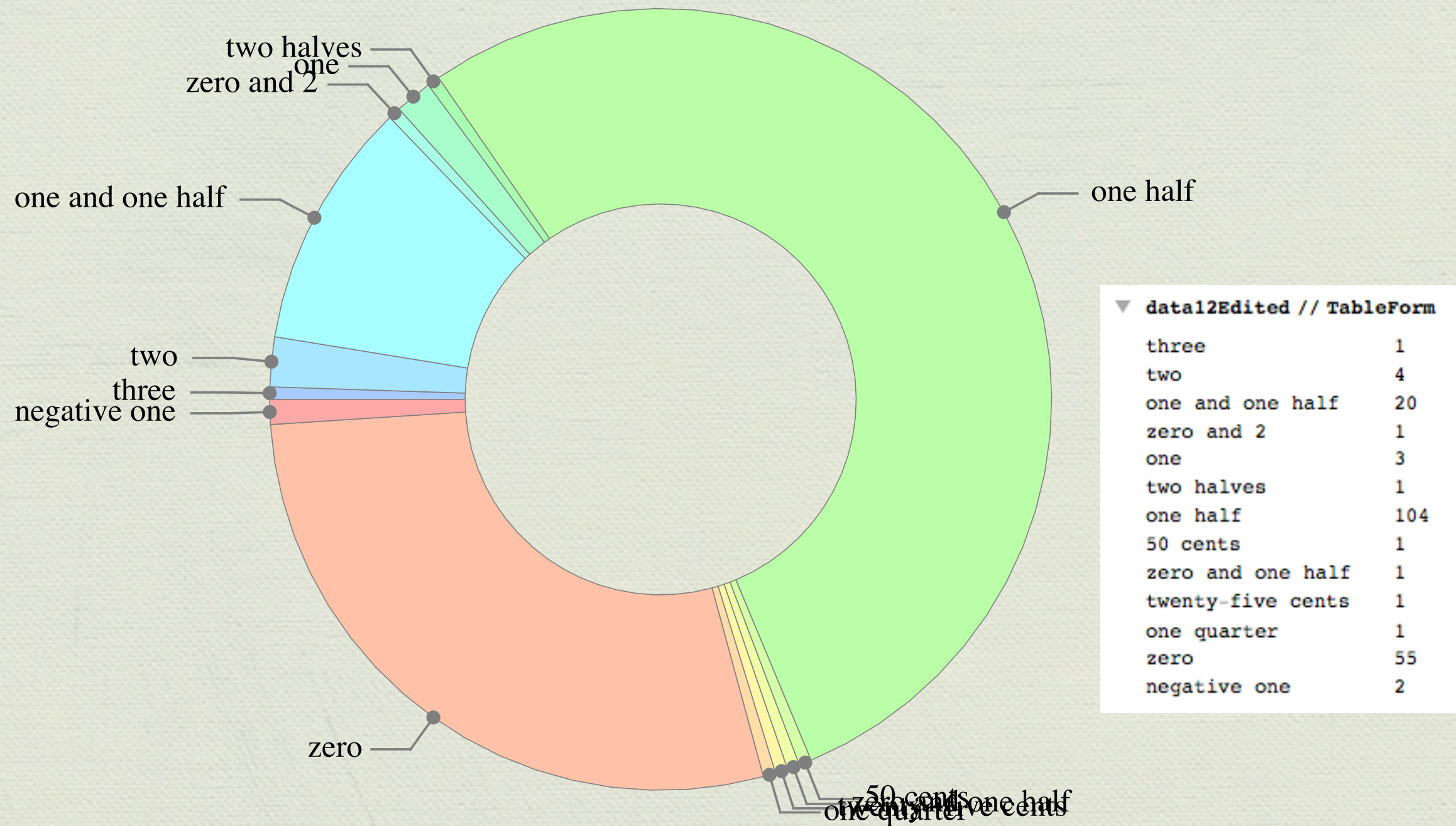
What number is half of one?

data2 // TableForm

incorrect	104
correct	104
given by interviewer	2
torn between two answers	4
"I don't know"	14
done	18
	6
"does not exist"	4
blank	2

What number is half of one?

data12Edited // TableForm	
three	1
two	4
one and one half	20
zero and 2	1
one	3
two halves	1
one half	104
50 cents	1
zero and one half	1
twenty-five cents	1
one quarter	1
zero	55
negative one	2

Tuesday, March 9, 2010

4058:I don't think it could be half of one. I don't think there's half of one. Can only be a half if it's even. For example 5 is half of 10, it kind of like doubles.
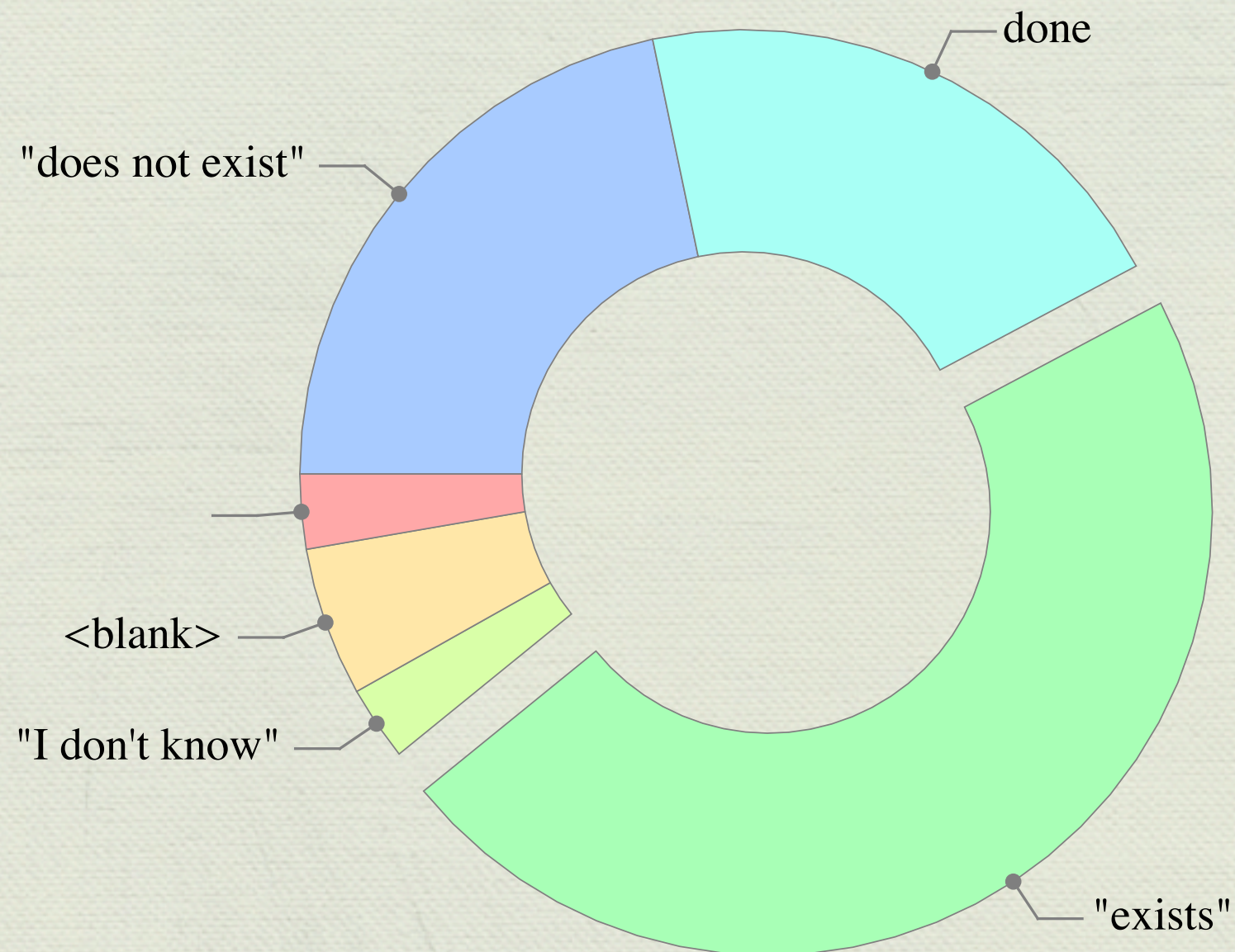
- Rnd 2889: "half is as small as you can go"

4545: "Zero is a number and if you're going to count backwards that's where you end and if you're going to count forward, that's where you end."
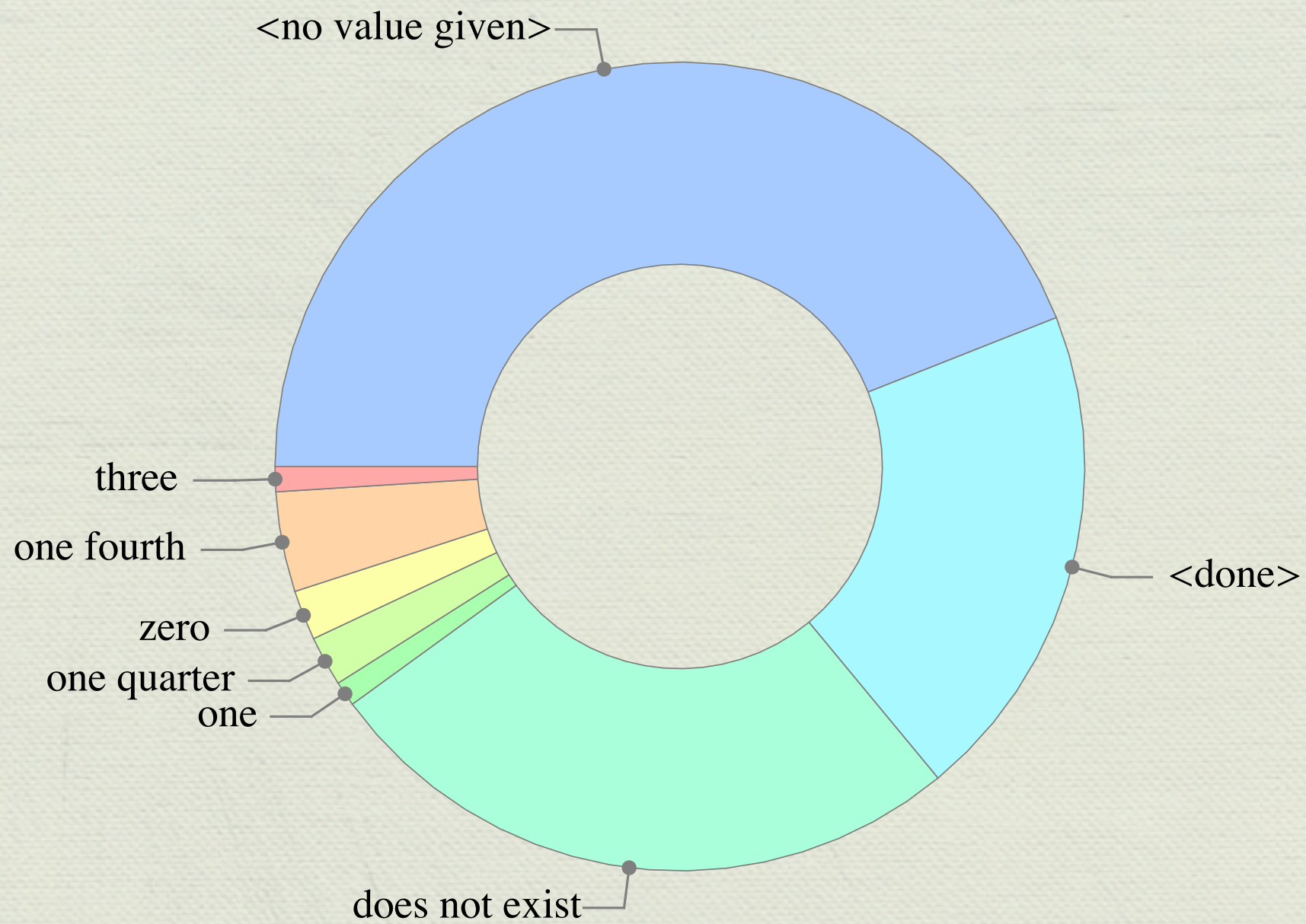
Is a number half of one−half?

done

"does not exist"

"exists"

<blank>

"I don't know"

data3 // TableForm	
"does not exist"	56
done	53
"exists"	121
"I don't know"	7
<blank>	14
	7

Is some number half of one−half?

- <no value given>
- <done>
- three
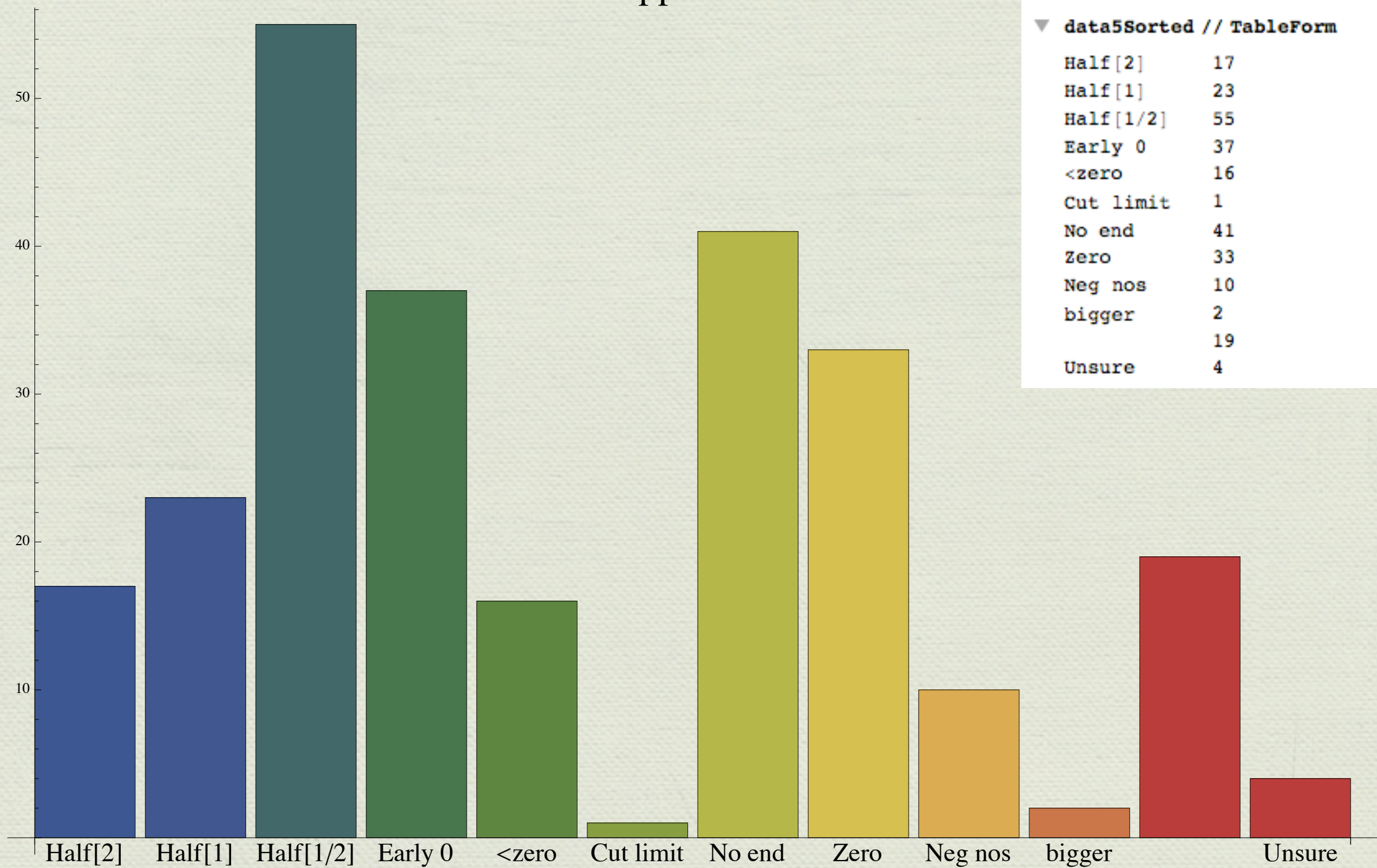- one fourth
- zero
- one quarter
- one
- does not exist

Rnd 2597: [reaches zero]: "because eventually the number would be so small, you can't cut it anymore. If you did, there'd be nothing left."

Stopped at...?


```
▼  data5Sorted // TableForm
   Half[2]      17
   Half[1]      23
   Half[1/2]    55
   Early 0      37
   <zero        16
   Cut limit    1
   No end       41
   Zero         33
   Neg nos      10
   bigger       2
                19
   Unsure       4
```

5509: "Each time, you're taking away a half."

- 6543: "Every time you are cutting in half you are getting more of a number but the number is less. So like half is less than 1 but half is also bigger than 1/4. It keeps on going on but then the numbers that were less get bigger because the numbers get smaller... it's sort of confusing."

- Rnd 4101:

- "Every time you keep on breaking it smaller and smaller until it will eventually... will have nothing. Anything you cut will eventually be zero, if you keep cutting it."

6117: [fake numbers]"Like if you have half and take away one third, and you'll have noting left. "

Rnd #: 2584 (gets to zero): "Because if you're going from a fourth and you minus another fourth, it will be nothing. "[Taking half as subtracting…]

6787: [getting bigger or smaller?] Smaller [how know?] No wait they get bigger, because of the fractions, wait smaller. A fourth is bigger than an eighth.

Rnd 2612:[reaches zero because] " It could keep going on... I think zero could be an answer because you'd have to know all the numbers to know what comes next. Zero could be the answer, but it's probably going to come up soon because hardly anyone knows every single answer. (You mean people wouldn't know which number comes next?) Yes."

- 3916: "When the numbers get bigger, the less it is.

- (I: Can you give an example?)

- 1/12 is a little, bec. you have so many pieces. You're not supposed to have so many pieces. If you have 12 pieces, that's good, bec. it's one whole.

- 6963: [littlest number]: For T8_3c1 said "Yes, 1/4" for T8_3c2 said "No, 1/4 is the littlest

7119:Because we start out with four and then you go to two, then one, then zero. So you're counting down.
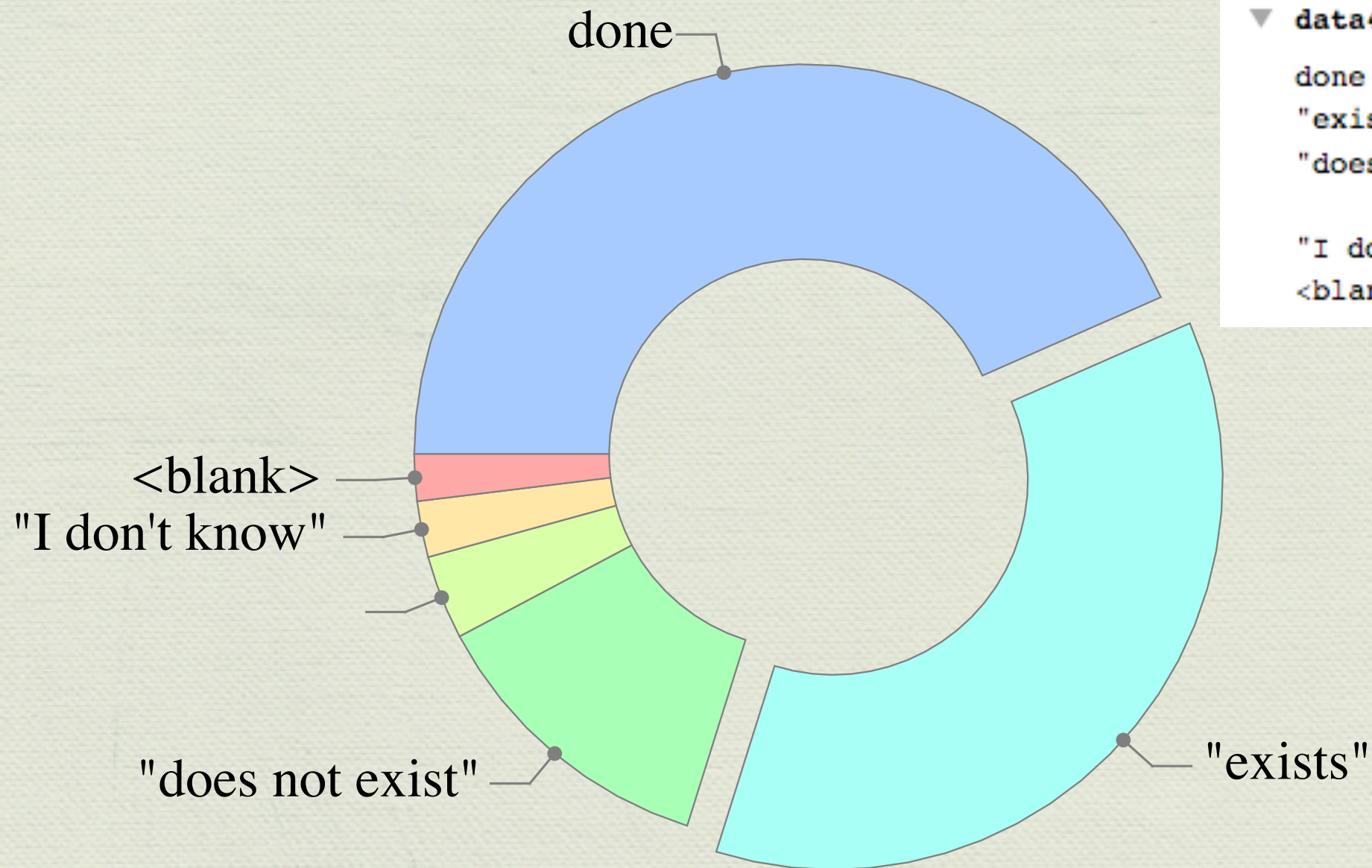
7606: "Because you're going to just keep getting smaller. It's probably going to get into the millions and keep going because you could double all the denominators and leave the numerator at one. The bigger the denominator the smaller the piece."

Summary Slides

Additional cases...(below 1/4)

data4 // TableForm

done	112
"exists"	94
"does not exist"	32
	9
"I don't know"	6
<blank>	5